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Reliability Assessment of Systems with Limited Information

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Abstract:
In our developed societies, engineering systems are characterized by a rapid growth in scale and complexity. The amount of information needed to model these systems with their complexity is, thus, growing as well. In contrast to this increasing need for information the available information remains almost at the same level. Hence, with increasing scale and complexity the gap between required and available information is growing quickly, so that uncertainties and risks are involved in our models and analyses to a greater extent than ever before. We address this challenge with concepts of imprecise probabilities for reliability assessment of engineering systems when only limited information is available. In order to achieve high numerical efficiency, in particular when dealing with large complex systems, the concept of survival signature is adopted for the reliability assessment. Based on the developments of a survival analysis and importance analysis of systems with multiple types of components from it is shown how imprecise probabilities help to reveal the most critical components of the system and the most critical uncertainties, as well. Conclusions on a targeted reduction of imprecision are drawn.
Key Words: systems reliability; imprecise probabilities; survival signature

1. Imprecise probabilities for generalized uncertainty modeling

Since our engineering systems are, to a significant extent, critical for the functionality of our economic and societal life, they require proper approaches and measures to verify and ensure their reliable performance. Reliability and performance analysis become increasingly complicated due to the growing uncertainties through complexity. The realistic quantification of uncertainties and their numerically efficient processing in complex analyses are, thus, the two key challenges in this context.

The uncertainty that is induced by limited and vague information represents epistemic uncertainty. Advancements in generalized uncertainty modeling are made to enable the quantification of epistemic uncertainties in form of an optimum compromise solution in the balance between three goals: (i) the complete representation of available information in the theoretical uncertainty model, (ii) the modeling without assumptions, which cannot be justified and potentially introduce artificial information, and (iii) the most appropriate modeling in view of the purpose of the analysis in order to provide the best possible basis for informed decisions. Clearly, the first consideration should be devoted to probabilistic modeling, naturally through subjective probabilities, which express a belief of the expert and can be integrated into a fully probabilistic framework in a coherent manner via a Bayesian approach. While this pathway is widely accepted and recognized as being very powerful, the potential of set-theoretical approaches and imprecise probabilities has only been utilized to some limited extent. Those approaches, however, attract increasing attention in cases when available information is not rich enough to specify subjective probability distributions [2]. Imprecise probabilities provide a significantly increased model flexibility through a combination of set-theoretical models with probabilistic models and keep the nature of the available information consistent throughout the entire analysis.

The conceptual basis for imprecise probabilities is to distinguish between probabilistic subjectivity and imprecision as different forms of epistemic uncertainty, which provides a pragmatic criterion for classifying nondeterministic phenomena according to the nature of information. From this perspective, aleatory uncertainty (stochastic variation) and the subjective probabilistic form of epistemic uncertainty can be summarized as probabilistic uncertainty, whereas imprecision refers to the non-probabilistic form of epistemic uncertainty. This classification helps to avoid confusion if uncertainty appears as both probabilistic and non-probabilistic phenomena simultaneously in an analysis. An illustrative example for this situation is a random sample of imprecise perceptions (e.g., intervals due to limited measurement accuracy) of a physical quantity. While the scatter of the realizations of the physical quantity possesses a probabilistic character (frequentist or subjective), each particular realization from the population exhibits, additionally, imprecision with a non-probabilistic character. If an analysis involves this type of hybrid information, it is imperative to consider imprecision and probabilistic uncertainty simultaneously but not to mix the characteristics, so that imprecision is not described in terms of a probabilistic model and vice versa.

A mathematical framework for an analysis of this type has been established with imprecise probabilities. A key feature of imprecise probabilities is the identification of bounds on probabilities for events of interest; the uncertainty of an event is characterized with two values; a lower probability and an upper probability. The distance between the lower and upper probability bounds reflects the indeterminacy in model specifications expressed as imprecision of the models. This imprecision results from not introducing artificial model assumptions. It is described by implementing set-valued descriptors in the specification of a probabilistic model. The model description is thereby limited to an appropriate domain, and no further specific characteristics are ascribed. This introduces significantly less information in comparison with a specific subjective distribution function as used in a Bayesian approach. Imprecision in the model description expressed in a set-theoretical form is not translated into probabilities; it is not described in terms of probabilities, instead, it is reflected in the result as a set of probabilities which covers all plausible cases of model assumptions. This feature is particularly important when the calculated probabilities provide the basis for critical decisions. With imprecise probabilities the analysis may be performed with various relevant models to obtain a set of relevant results and associated decisions. This helps to avoid wrong decisions due to artificial restrictions in modeling.

The most straightforward approach to set up an imprecise probabilistic model is to identify respective set-valued distribution parameters. But the capabilities of the modeling are not limited to this approach. Imprecise probabilities are also capable of dealing with imprecise conditions, with imprecise dependencies between random variables, and with imprecise structural parameters and model descriptions [5, 6]. Further, multivariate models and statistical estimations and tests with imprecise sample elements can be constructed, results from robust statistics in the form of solution domains of statistical estimators can be considered directly. Recent overviews on imprecise probabilities with applications in engineering are provided in [2] and [1]. Subsequently, we use imprecise probabilities for reliability assessment of engineering systems when only limited information is available.

2. System reliability analysis

In order to address the issue of numerical efficiency, in particular when dealing with large complex systems, the concept of survival signature [3] is adopted for the reliability assessment. Suppose there is a system with m components. Let the state vector of the components be $\underline{x} = (x_1, x_2, \cdots, x_m) \in \{0, 1\}^m$ with $x_i = 1$ if the i^{th} component is in working state and $x_i = 0$ if not. $\emptyset = \emptyset(\underline{x}) : \{0,1\}^m \to \{0,1\}$ defines the system structure function, i.e., the system status based on all possible $\underline{x}$. $\emptyset$ is 1 if the system is in a functional condition for state vector $\underline{x}$ and 0 if not. Now consider a system with $K \geq 2$ types of m components, with m_k indicating the number of components of each type and $\sum_{k=1}^{K} m_k = m$. It is assumed that the failure times of the same component type are independently and identically distributed, or exchangeable. The survival signature becomes $\emptyset(l_1, l_2, \ldots, l_k)$, with $l_k = 0,1, \ldots, m_k$ for $k = 1,2, \ldots, K$. There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with precisely l_k components x_i^k equal to 1, so with $\sum_{i=1}^{m} x_i^k = l_k$. Let $S_{l_1, l_2, \ldots, l_K}$ denote the set of all state vectors for the whole system for which $\sum_{i=1}^{m_k} x_i^k = l_k$, $k = 1,2, \ldots, K$. Assume that the random failure times of components of the different types are fully independent, and in addition the components are exchangeable within the group of components of the same type. The survival signature can then be written as:

$$\emptyset(l_1, \ldots, l_k) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right] \times \sum_{\underline{x} \in S_{l_1, \ldots, l_k}} \emptyset(\underline{x}) \ . \quad (1)$$

$C_k(t) \in \{0,1,\dots,m_k\}$ denotes the number of k components working at time t. Assume that the components of the same type have a known CDF, $F_k(t)$ for type k. Moreover, the failure times of different component types are assumed as independent of one another. Hence

$$P\left(\cap_{k=1}^{K}\{C_k(t)=l_k\}\right)=\prod_{k=1}^{K}P(C_k(t)=l_k)=\prod_{k=1}^{K}\binom{m_k}{l_k}[F_k(t)]^{m_k-l_k}[1-F_k(t)]^{l_k}, \quad (2)$$

and the survival function of the system with K types of components becomes

$$P(T_s>t)=\sum_{l_1=0}^{m_1}\dots\sum_{l_k=0}^{m_k}\emptyset(l_1,\dots,l_K)P\left(\cap_{k=1}^{K}\{C_k(t)=l_k\}\right). \quad (3)$$

The structure of Eq. (3) shows a separation of the structure of the system from the failure time distribution of its components, which facilitates a very efficient analysis. The survival signature needs to be calculated only once for the entire analysis.

On this basis the reliability of a system can be analysed via Monte Carlo simulation in a very efficient manner. The survival signature yields the probability that the system is in functional condition knowing the number of components for each type (i.e. $l_1, l_2, \dots, l_k$) that are working. This system is equivalent to a system with k components that can be in as many states as components of the respective component type exist. Therefore, the survival signature can be interpreted as the "production capability"' of the system. The simulation can then be performed based on the concept proposed in [7]. The key steps of the simulation are:

1) sample the transition times of the first component type, hence a sequence of transition times $t_1, t_2, \dots$ are obtained

2) repeat the procedure of step 1 for the next component types, one by one, which will yield further sequences of additional transition times

3) reorder all the transition times of $(t_1, t_2, \dots)$.

4) compute the probability that the system functions for each time interval by evaluating the survival signature

5) repeat steps 1 to 4 for n system histories and average the obtained results

In the result the probability of system survival over the time t is obtained.

3. Reliability assessment with imprecise information

A major challenge for reliability assessment of complex systems is the specification of the probabilistic models for the characterization of the system performance. The concept of imprecise probabilities enables a comprehensive and realistic mathematical modelling of vagueness, indeterminacy and imprecision when specifying these probabilistic models and a translation thereof into the results of the reliability analysis. Specifically, component parameters and parameters of probabilistic models for component performance can be described in form of value ranges with lower and upper bounds. This leads to associated value ranges with lower and upper bounds for the survival function of the system. This approach is demonstrated on the example system in Fig. 1.

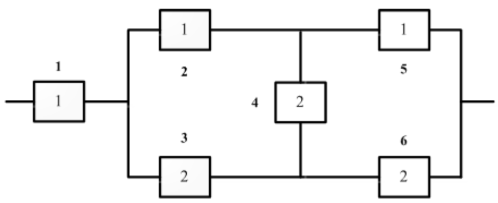

Figure 1. System with two types of components.

The component failure times are described with a Weibull distribution for component type 1 and with a Gamma distribution for component type 2. The distribution parameters are modelled as intervals, whereby the size of the intervals reflects the magnitude of missing information, see Table 1.

Table 1. Distribution parameters for system components with imprecision.

Component Type	Distribution Type	Parameters (α,β)
1	Weibull	([1.2,1.8], [2.3,2.9])
2	Gamma	([0.8,1.6], [1.3,2.1])

The Monte Carlo simulation to obtain the survival function of the system has now to capture an entire set of probabilistic problem descriptions. Out of the set of survival functions associated with this set of probabilistic problem descriptions only the bounding functions are of interest since these represent the best case and worst case results, in terms of probabilities, important for decision making. In addition, knowing the bounding functions, the range between these functions provides information about sensitivities of the system performance with respect to the magnitude of imprecision in the problem set up. For the numerical analysis it is thus sufficient to identify the bounding survival functions, which is realized by solving an optimization over the domain of the imprecise parameters. For the example system this leads to the bounding survival functions in Fig. 2.

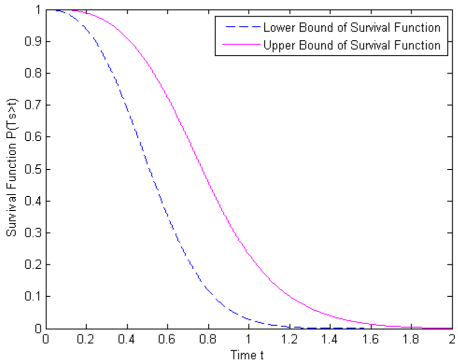

Figure 2. Lower and upper bounds of the survival function for the system in Fig. 1.

4. Conclusions

The concept of survival signature is a practical method for efficient and transparent reliability analysis of complex systems with multiple component types. It can be combined in a straight-forward manner with stochastic simulation approaches, which supports its general applicability. The efficiency of the approach is based on the feature that the system model needs to be analysed only once in the entire analysis, specifically to obtain the survival signature. An expansion to take into account limited and vague information has been achieved with the aid of imprecise probabilities. This allows to analyse problems characterised by both epistemic and aleatory uncertainties in one go. In conjunction with the developments of a survival analysis and importance analysis of systems with multiple types of components from [4] it is possible to utilize imprecise probabilities to reveal the most critical components of the system and the most critical uncertainties, as well. On this basis conclusions on a targeted reduction of imprecision can be drawn.

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